GCE AS MARKING SCHEME

SUMMER 2023

AS
FURTHER MATHEMATICS
UNIT 2 FURTHER STATISTICS A
2305U20-1

INTRODUCTION

This marking scheme was used by WJEC for the 2023 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCE AS FURTHER MATHEMATICS

UNIT 2 FURTHER STATISTICS A

SUMMER 2023 MARK SCHEME

Qu.	Solution	Mark	Notes
1 (a)	$\mathrm{E}(4Y - 2X + 1) = 4\mathrm{E}(Y) - 2\mathrm{E}(X) + 1$	M1	Use of
	$= 4 \times 10 - 2 \times 17 + 1$		
	$= 7$	A1	
(b)	$\mathrm{Var}(4Y - 5X + 3) = 4^2\mathrm{Var}(Y) + 5^2\mathrm{Var}(X)$	M1	
	$= 16 \times 16 + 25 \times 64$		
	$= 1856$	A1	cao
(c)	$\mathrm{E}(X^2) = \mathrm{Var}(X) + \big(\mathrm{E}(X)\big)^2$	M1	Use of
	$\mathrm{E}(X^2) = 64 + 17^2$		
	$\mathrm{E}(X^2) = 353$		
	$\mathrm{E}(X^2Y) = \mathrm{E}(X^2) \times \mathrm{E}(Y)$	M1	FT their $E(X^2)$ provided $\neq 17^2$
	$\mathrm{E}(X^2Y) = 353 \times 10$		
	$\mathrm{E}(X^2Y) = 3530$	A1	cao
		Total [7]	

Qu.	Solution	Mark	Notes
2(a)	$\overline{x} = 14$ $\bar{y} = 8$	B1	Both
	$b = \dfrac{20 - 8}{19 - 14}$	M1	
	$b = 2.4$	A1	convincing
	Equation of regression line is		
	$y - 20 = 2.4(x - 19)$	M1	oe
	$y = 2.4x - 25.6$ *ag	A1	convincing
	<u>1st ALTERNATIVE METHOD</u> $y = mx + c$		
	$20 = 2.4 \times 19 + c$	(M1)	oe
	$c = -25.6$		
	$y = 2.4x - 25.6$ *ag	(A1)	convincing
	<u>2nd ALTERNATIVE METHOD</u> $y = bx + a$ $20 = 19b + a$	(M1)	For 1st equation
	$\dfrac{240}{30} = \dfrac{420}{30}b + a$	(B1)	For sight of $\frac{240}{30}$ and $\frac{420}{30}$
		(A1)	For correct equations
	Solve simultaneously to get $a = -25.6$ and $b = 2.4$	(M1)	
	$y = 2.4x - 25.6$ *ag	(A1)	
	When $x = 26$		
	$y = 2.4 \times 26 - 25.6$ $y = 36.8$	B1	
(b)	Comment on linearity e.g. Scatter diagram may not be linear. Correlation might be weak.	E1	
	Comment on range of x values. e.g. 26 may be beyond the range of observed x values.	E1	
		Total [8]	

Qu.	Solution	Mark	Notes
3(a)	(Let the random variable X be the lifetime in years of a hair dryer.) $\lambda = \frac{1}{2} = 0.5$	B1	
	$\mathrm{P}(1.8 \le X \le 2.5) = \int_{1.8}^{2.5} 0.5\mathrm{e}^{-0.5x}\mathrm{d}x$	M1	FT their λ for M1A1 Alternative M1A1 $\mathrm{e}^{-0.5\times1.8} - \mathrm{e}^{-0.5\times2.5}$
	$= [-\mathrm{e}^{-0.5x}]_{1.8}^{2.5}$	A1	Limits required
	$= 0.1201$	A1	cao, 3sf required
	OR		
	$\mathrm{P}(1.8 \le X \le 2.5) = F(2.5) - F(1.8)$	(M1)	use of
	$= (1 - \mathrm{e}^{-0.5\times2.5}) - (1 - \mathrm{e}^{-0.5\times1.8})$	(A1)	Must be cdf
	= 0.1201	(A1)	cao, 3sf required
(b)			FT their λ for M1M1
	$P(X \ge k) = 0.2$ or $P(X < k) = 0.8$	M1	Attempt to use of
	$1 - \mathrm{e}^{-0.5k} = 0.8$ $\mathrm{e}^{-0.5k} = 0.2$ $-0.5k = \ln 0.2$	M1	
	$k = 3.2$	A1	cao, oe (eg. 2ln5)
	OR		
	$\mathrm{P}(X < k) = \int_0^k 0.5\mathrm{e}^{-0.5x}\mathrm{d}x = 0.8$	(M1)	Or integrating between k and ∞ and setting $= 0.2$
	$[-\mathrm{e}^{-0.5x}]_0^k = 0.8$ $\left(-\mathrm{e}^{0.5k}\right) - (-1) = 0.8$ $0.2 = \mathrm{e}^{-0.5k}$		
	$\ln 0.2 = -0.5k$	(M1)	
	$k = 3.2$	(A1)	cao

Qu.	Solution	Mark	Notes
3(c)	Let the random variable Y be the number of hair dryers replaced in 5 years. $\text{Po}(2.5)$ over 5 years	B1	
	$\text{P}(Y > 3) = 1 - \text{P}(Y \leq 3)$	M1	FT their 2.5 for discrete distribution
	$\text{P}(Y > 3) = 1 - 0.75758$		
	$\text{P}(Y > 3) = 0.24242$	A1	cao
(d)	Valid assumption e.g. Jon doesn't think the quality is so poor that he buys a different brand. The hair dryers bought weren't from a faulty batch. The quality of the hair dryers hasn't improved in five years. Manufacturing methods haven't changed. The hair dryers **still** last on average 2 years. The replacement is a new hair dryer (not a used one).	E1	
		Total [11]	

Qu.	Solution	Mark	Notes
4(a)	$\frac{1}{480}b^4 + \frac{7}{15} = 1$	M1	M1 for setting $F(b) = 1$
	$b^4 = 256$		
	$b = 4$ ***ag**	A1	At least one step between M1 and $b = 4$ Convincing.
(b)	$\mathrm{P}(X < 2.5) = F(2.5)$ $= \frac{1}{480} \times 2.5^4 + \frac{7}{15}$	M1	Attempt to substitute 2.5 into $F(x)$.
	$= 0.548$ or $\frac{1403}{2560}$	A1	3sf or better
(c)	Lower quartile = 1	B1	
(d)	$\frac{1}{480}u^4 + \frac{7}{15} = 0.75$	M1	
	$u^4 = 136$	A1	
	$u = 3.41$	A1	
(e)	$\mathrm{P}(X > 3.5) = 1 - F(3.5)$	M1	si
	$= 1 - \left(\frac{1}{480} \times 3.5^4 + \frac{7}{15}\right)$		
	$= 1 - 0.779 \ldots = 0.2207 \ldots.$	A1	
	$P(X < k) = 0.2207 \ldots..$		
	$\frac{1}{4}k = 0.2207 \ldots.$	M1	si FT their 0.2207… provided > 0.125 and < 0.5
	$k = 0.883$	A1	cao Do not allow 0.884. $\left(k = \frac{113}{128}\right)$
		Total [12]	

Qu.	Solution	Mark	Notes
5 (a)	Two appropriate circumstances. e.g. When the data are ordinal. e.g. When the data are not bivariate normal.	B2	B1 for one circumstance.
(b)(i)	H_0: there is no association between the rank given by the farmer and the price. H_1:there is an association between the rank given by the farmer and the price.	B1	Both. Do not allow correlation.
	5% two tailed critical value = ($\pm$)0.8286	B1	
	Since $0.9429 > 0.8286$ there is sufficient evidence to reject H_0.	B1	Comparing TS to CV FT their CV
	There is sufficient evidence to suggest that there is an association between the rank given by the farmer and price.	E1	Comment in context Only award if previous 3 B1 have been awarded
(ii)	(table below)	B1	Correct values for one set of PTO ranks

Tractor	Farmer's rank	PTO rank (highest to lowest)	PTO rank (lowest to highest)
A	1	4	3
B	6	3	4
C	5	5	2
D	4	6	1
E	2	1	6
F	3	2	5

Qu.	Solution	Mark	Notes
	$\sum d^2 = 24$ or $\sum d^2 = 46$	B1	si
	$r_s = 1 - \frac{6\times24}{6\times35}$ or $r_s = 1 - \frac{6\times46}{6\times35}$	M1	FT their '24' or '46'
	$r_s = \pm0.3143$ or $\pm\frac{11}{35}$	A1	
(iii)	Two valid comment. e.g. The PTO variable isn't very strongly associated with the farmer's ranks so may not be worth analysing. e.g. The salesman doesn't need to work that hard because the farmer already prefers the most expensive tractors. e.g. because the number of tractors in the sample is small, the salesman should not place too much reliance on the results.	E2	
		Total [12]	

Qu.	Solution	Mark	Notes
6 (a)	H_0: The data can be modelled by the binomial distribution $\text{B}(20, 0.1)$. H_1: The data cannot be modelled by the binomial distribution $\text{B}(20, 0.1)$.	B1	or equivalent, must state B(20,0.1)
(b)(i)	Expected frequencies are $(f = (\text{P}(X = 0) \times 110)$ $f = 13.37$ $(g = (\text{P}(X = 2) \times 110)$ $g = 31.37$	 B1 B1	 Accept 13.4 Accept 31.4
(ii)	Combine classes with expected frequencies less than 5	M1	SC for solution that does not combine classes. (M0M1m1A0B1B1 m1A0)

Number of boats not taken out.	0	1	2	3	4 or more
Obs	10	35	29	25	11
Exp	13.37	29.72	31.37	20.91	14.63

Qu.	Solution	Mark	Notes
	Use of χ^2 stat = $\sum\frac{(O-E)^2}{E}$ OR $\sum\frac{O^2}{E} - N$	M1	Must see at least 2 terms added
	$= \frac{(10-13.37)^2}{13.37} + \frac{(35-29.72)^2}{29.72} + \frac{(29-31.37)^2}{31.37} + \frac{(25-20.91)^2}{20.91} + \frac{(11-14.63)^2}{14.63}$	m1	$= \frac{10^2}{13.37} + \frac{35^2}{29.72} + \frac{29^2}{31.37} + \frac{25^2}{20.91} + \frac{11^2}{14.63} - 110$
	= 3.667…	A1	cao 3.66760 if unrounded values used.
	DF = 4	B1	
	5% CV = 9.488	B1	Accept other test levels 1% CV = 13.277 10% CV = 7.779
	Since $3.667 < 9.488$ there is insufficient evidence to reject H_0.	m1	Dep on 2nd M1
	Insufficient evidence to reject the suggestion that the number of boats not taken out each day can be modelled by the binomial model B(20, 0.1)	A1	cso provided B1 awarded in (a) A0 for categorical statements

Qu.	Solution	Mark	Notes
6(c) (i)	Let the random variable Y be the number groups that turn up expecting to take a boat out. $Y \sim \mathrm{B}(22,0.9)$	B1	May be seen or implied in (ii)
(ii)	Let the random variable S be the income of the company in pounds. Values for s are 330, 310 and 290	B1	All 3 values and no others
	$\mathrm{P}(S = 330) = \mathrm{P}(Y \leq 20)$ $= 0.6608$ $\mathrm{P}(S = 310) = \mathrm{P}(Y = 21)$ $= 0.2407$ $\mathrm{P}(S = 290) = \mathrm{P}(Y = 22)$	B1	Recognising link between income and the probability of the number of groups who turn up to take a boat, si If using $W \sim \mathrm{B}(22,0.1)$, probabilities are $\mathrm{P}(W \geq 2) = 0.6608$ $\mathrm{P}(W = 1) = 0.2407$ $\mathrm{P}(W = 0) = 0.0985$
	$= 0.0985$	B2	B2 for three correct probabilities. B1 for one correct probability.
	$\mathrm{E}(S) = 330 \times 0.6608 + 310 \times 0.2407 + 290 \times 0.0985$	M1A1	M1 for one correct term and addition.
	$= £321.25$	A1	
	<u>Alternative solution</u> Let the random variable T be the loss of the company in pounds. Values for t are $(0,)$ 20 and 40	(B1)	All 3 values and no others.
	$\mathrm{P}(T = 0) = \mathrm{P}(Y \leq 20)$ $= 0.6608$ $\mathrm{P}(T = 20) = \mathrm{P}(Y = 21)$ $= 0.2407$ $\mathrm{P}(T = 40) = \mathrm{P}(Y = 22)$	(B1)	Recognising link between loss and the probability of the number of groups who turn up to take a boat, si If using $W \sim \mathrm{B}(22,0.1)$, probabilities are $\mathrm{P}(W \geq 2) = 0.6608$ $\mathrm{P}(W = 1) = 0.2407$ $\mathrm{P}(W = 0) = 0.0985$
	$= 0.0985$	(B2)	B2 for three correct probabilities. B1 for one correct probability.
	$\mathrm{E}(T) = (0 \times 0.6608 +)\ 20 \times 0.2407 + 40 \times 0.0985$ = £8.75	(M1A1)	M1 for one correct term and addition.
	Net income (= £330 - £8.75) = £321.25	(A1)	
(d)	Valid reason. e.g. The manager is justified because the expected income is greater than $£15 \times 20 = £300$	E1 **Total** **[20]**	Accept “Not justified” with appropriate reason such as “not a long term strategy because it will harm the brand.” FT their E(S)